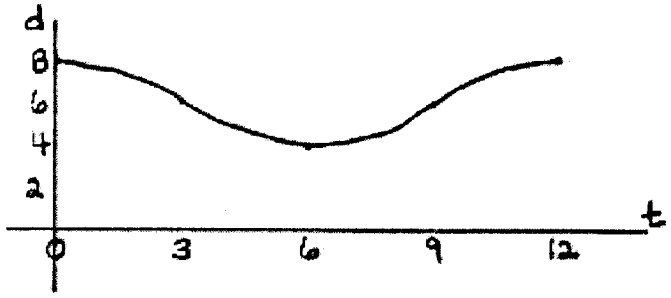
MCR 3U1 **Trig Applications – Solutions**

1. a) Let t be the time (in hrs) since 12:00pm
Let d be the depth of the water (m).

Given:

t	d
0	8m
6	4m
12	8m



Since the graph starts at high tide, we will model the equation with $y = a\cos kx$

Amplitude:
$|a| = \frac{8-4}{2}$
$= 2$

period = 12 hrs
$12 = \frac{360}{k}$
$12k = 360$
$k = 30$

Vertical shift
∵ average depth = 6m
⇒ the graph is shifted up 6

∴ Equation is $d = 2\cos(30t) + 6$

b) At 8 am, $t = 20 \Rightarrow d = 2\cos[30(20)] + 6$

$d = 2\cos(600°) + 6$
$d = 5$

c) When $d = 6.3$m $\Rightarrow$ $6.3 = 2\cos(30t) + 6$

$6.3 - 6 = 2\cos(30t)$
$0.3 = 2\cos(30t)$
$0.15 = \cos(30t)$
$\cos^{-1}(0.15) = 30t$

$t = \frac{\cos^{-1}(0.15)}{30}$

c) Note that $\cos^{-1}(0.15)$ yields 2 possible angles:

In degrees: $\cos^{-1}(0.15) = 81.37°$ OR $360 - 81.37 = 278.63°$

$\therefore t = \frac{81.37}{30} \doteq 2.7$ or $t \doteq \frac{278.63}{30} \doteq 9.3$

$\therefore$ There are two times when the water depth is 6.3 m:

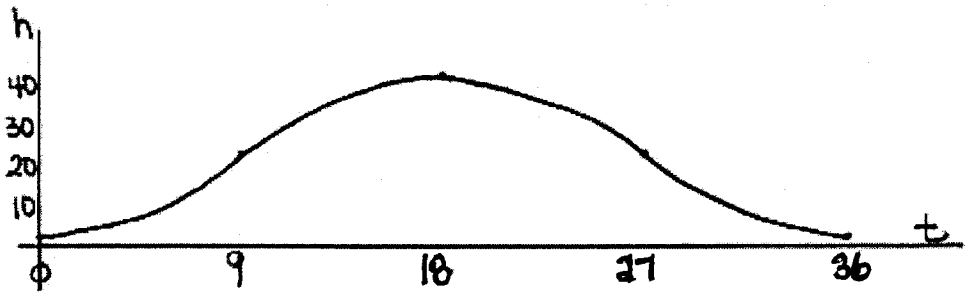
i) after 2.7 hrs (ie. at 2:42 pm)

2) after 9.3 hrs (ie. at 9:18 pm)

2. a) Let t be the time since getting on the Ferris Wheel.
Let h be the height above the ground. (metres)

Given:

t	h
0	2m
9	22 m
18	42 m
27	22m
36	2m



Starting with $h = -a\cos kt$, (reflection of $y = a\cos kt$)

$|a| = \frac{42-2}{2} = 20$

period $= 36$ s

$36 = \frac{360}{k}$

$36k = 360$

$k = 10$

Vertical shift:

up $\frac{42+2}{2} = \frac{44}{2} = 22$

$\Rightarrow$ the graph is shifted up 22 units

$\therefore$ equation is $h = -20\cos(10t) + 22$

Starting with $h = \sin t$

$|a| = 20$ period $= 36$ s, $k = 10$ vertical shift: up 22 units

phase shift: graph has been shifted right 9 units

$\therefore$ Equation is $h = 20 \sin[10(t-9)] + 22$

2. b) After 3 s, $h = -20 \cos[10(3)] + 22$

$h \doteq 4.7$ m

c) When $h = 28$ m, $28 = -20 \cos(10t) + 22$

$6 = -20 \cos(10t)$

$-0.3 = \cos(10t)$

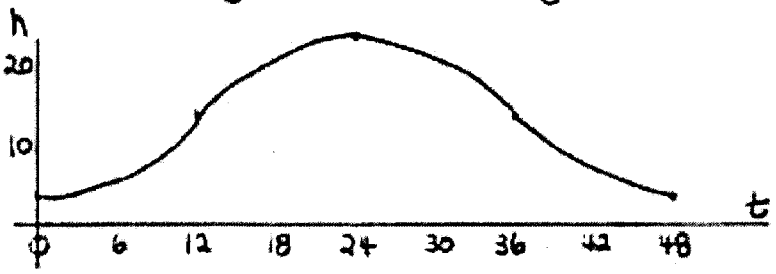
$t = \dfrac{\cos^{-1}(-0.3)}{10}$

$t = 10.7, 25.3$

$\uparrow (36 - 10.7)$

3 a) Let t be the time since getting on the Ferris Wheel.
Let h be the height above the ground (metres).

t	h
0	3 m
12	13m
24	23 m
36	13m
48	3m



Start with base equation: $y = -a\cos kt + q$

$|a| = \frac{23-3}{2}$
$= 10$

period $= 48$ s.
$\therefore 48 = \frac{360}{k}$
$48k = 360$
$k = \frac{15}{2} = 7.5$

Vertical shift:
up $\frac{23+3}{2} = \frac{26}{2} = 13$ units

$\therefore$ equation is $h = -10\cos(7.5t) + 13$

As a sine function, the graph has a phase shift right 12 units.

$\therefore$ equation is $h = 10\sin[7.5(t-12)] + 13$

b) After 2.5 min = 150 seconds, $h = -10\cos[7.5(150)] + 13$

$h = 5.9$ m

c) When $h = 9$ m, $9 = -10\cos(7.5t) + 13$

$-4 = -10\cos(7.5t)$

$0.4 = \cos(7.5t)$

$t = \frac{\cos^{-1}(0.4)}{(7.5)}$

$t \doteq 8.9$ s, 39.1 s.
$\uparrow (48 - 8.9)$

4. Let t be the time since the object starts oscillating.
Let d be the distance from the middle position.

$|a| = \frac{30}{2} = 15$ period $= \frac{4}{5}$ Vertical shift up 15 units

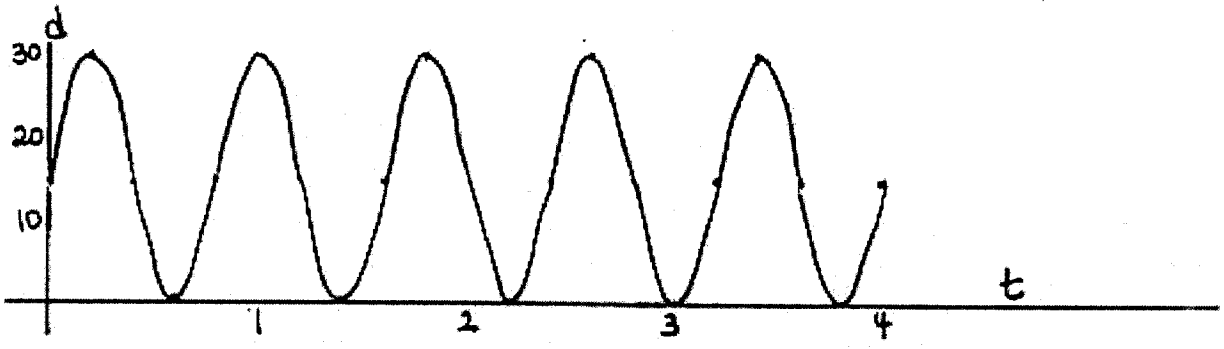
$\frac{4}{5} = \frac{360}{k}$

$4k = 1800$

$k = 450$

∴ Equation is

$d = 15 \sin(450t) + 15$



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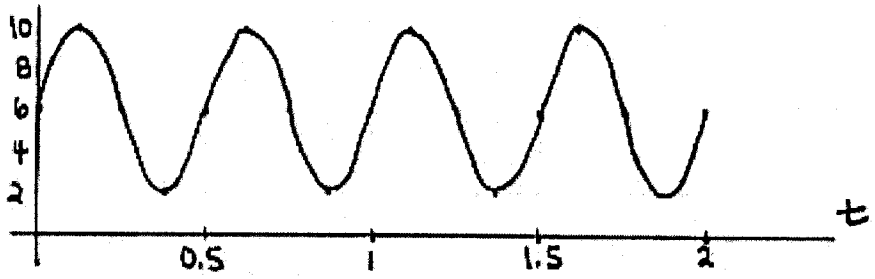

$|a| = \frac{10-2}{2}$ period $= \frac{2}{4} = \frac{1}{2}$ Vertical shift up 6 units

$= \frac{8}{2} = 4$

$\frac{1}{2} = \frac{360}{k}$

$k = 720$

∴ Equation is $y = 4 \sin(720t) + 6$